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A CONSISTENT FUZZY EOQ MODEL USING SIGNED DISTANCE DEFUZZIFICATION: MATCHING ORDER QUANTITY AND TOTAL COST ACROSS FUZZY AND CRISP DOMAINS

C. Venkatesan* & J. Vimala**

* Professor, Department of Mathematics, Dhanalakshmi Srinivasan Engineering College, Perambalur, Tamil Nadu, India

** Assistant Professor, Department of Mathematics, Srinivasan College of Arts and Science, Perambalur, Tamil Nadu, India

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Abstract:

This paper develops a fuzzy inventory model under uncertainty using triangular fuzzy numbers and the signed distance defuzzification method. A key contribution is the derivation of exact conditions under which the economic order quantity (EOQ) and total cost (TC) remain consistent before and after defuzzification. The model is validated numerically, and implementation is demonstrated in Python with visualizations. This approach offers operational clarity and mathematical consistency in fuzzy decision-making models.

Key Words: EOQ, Fuzzy Inventory Model, Signed Distance, Defuzzification, Triangular Fuzzy Number, Total Cost, Python

Introduction:

Inventory management is a foundational aspect of operations research and supply chain optimization, with the Economic Order Quantity (EOQ) model playing a pivotal role in balancing ordering and holding costs. Traditionally, EOQ models assume that all parameters such as demand, ordering cost, holding cost, and lead time are known precisely. However, in real-world decision-making environments, these parameters often exhibit inherent uncertainty due to fluctuating market demands, ambiguous supplier information, incomplete historical data, or managerial subjectivity. Modelling this imprecision through classical probability may not always be appropriate, especially when statistical distributions are unknown or ill-defined.

Fuzzy set theory, introduced by Zadeh in 1965, provides an effective framework for modelling such uncertainties. In a fuzzy EOQ model, imprecise parameters are represented as fuzzy numbers, enabling decision-makers to incorporate vagueness and linguistic terms directly into the model. However, the key challenge lies in interpreting and defuzzifying fuzzy outputs particularly the optimal order quantity and total cost in a manner that preserves consistency and practical relevance.

Among the various defuzzification techniques, the signed distance method has emerged as a robust and intuitive approach. Unlike centroid-based methods which may obscure the direction and spread of uncertainty, the signed distance method considers both the magnitude and orientation of fuzziness. It provides a linear and interpretable way to translate fuzzy numbers into crisp values, making it particularly suitable for optimization-based models where monotonicity and consistency are essential. This study proposes a consistent fuzzy EOQ model using signed distance defuzzification, with a focus on ensuring that the derived optimal order quantity and total cost from the fuzzy model closely match their crisp counterparts**.** This consistency is critical for decision-makers who seek to rely on fuzzy modelling without facing abrupt or unrealistic deviations in operational planning.

The novelty of this work lies in two key contributions:

- Mathematical consistency: By applying the signed distance defuzzification method to triangular (or trapezoidal) fuzzy numbers used in the EOQ model, we demonstrate that the optimal values derived from the fuzzy framework can align with those from the crisp domain under well-defined conditions.
- Practical interpretability: The proposed model ensures that fuzzy and crisp solutions are not only theoretically linked but also interpretable and usable in real-time inventory decision-making. This helps bridge the gap between abstract fuzzy logic and applied operational strategies.

Moreover, this study validates the approach through comparative numerical experiments, showing that the signed distance method yields stable, intuitive, and computationally consistent results. The resulting framework allows practitioners to maintain flexibility in parameter estimation while ensuring reliability in cost-based optimization decisions.

Literature Review:

The EOQ model, originally formulated by Harris (1913), assumes that all parameters involved in inventory decisions are precisely known. However, in practical situations, variables such as demand, holding cost, and ordering cost are often imprecise or uncertain. To address this, fuzzy set theory, introduced by Zadeh (1965), has been increasingly applied in inventory modelling. Park (1987) pioneered the use of fuzzy numbers in EOQ models by replacing crisp cost components with triangular fuzzy numbers. Chang (1996) further extended this by proposing models incorporating fuzzy demand and ordering costs, relying on fuzzy arithmetic and defuzzification methods.

A crucial component of fuzzy modelling is defuzzification, which converts fuzzy outputs into crisp values for decision-making. Several methods have been explored in the literature, including the centroid method (Zimmermann, 1991), mean of maxima (Dubois and Prade, 1980), graded mean integration representation (Cheng, 1998), and the signed distance method (Yager, 1981; Tran and Duckstein, 2002). Among these, the signed distance method has gained prominence for its ability to preserve the directionality and distribution characteristics of fuzzy numbers.

Researchers have also studied the consistency between fuzzy-derived and crisp-derived solutions. Celik et al. (2009) and Jain et al. (2012) highlighted that discrepancies between fuzzy and crisp EOQ models can lead to impractical or non-intuitive inventory decisions. More recent works, such as those by Latha and Karunambigai (2016), and Dinesh and Vijayaragavan (2020), have adopted trapezoidal fuzzy numbers with centroid-based defuzzification techniques, though these methods sometimes produce results inconsistent with classical models.

The signed distance defuzzification approach addresses these inconsistencies by providing a more linear and interpretable transition from fuzzy to crisp values. Studies by Goswami and Roy (2011) and Das et al. (2020) demonstrate that using signed distance techniques allows for consistent EOQ modelling in fuzzy environments, particularly where certain parameters exhibit vagueness. Despite these advancements, limited work has focused on explicitly matching fuzzy and crisp EOQ outcomes. This research aims to bridge that gap by proposing a fuzzy EOQ model that uses signed distance defuzzification to ensure consistent and practical results across both fuzzy and deterministic domains.

Notations:

Q	-	Order Quantity
I	-	Investment required to reduce the lost sales fractions
α	-	Annual fractional cost of capital investment
s	-	Expected stock out
P	-	Safety factor
h	-	Holding cost per unit per year
E	-	Demand
ER	-	Lead time per week
E (X-W)	-	Expected shortage quantity at the end of the cycle
C (1-t)	-	Lead time crashing cost

Inventory Model:

This model constitutes a periodic review model. The total cost is

$$TC = \alpha I + \frac{1}{Q}[s + C(1-t)] + h\left[P - ER + \frac{EQ}{2} + \alpha E(X - W)\right]$$

Differentiate partially with respect to Q, we get

$$\frac{\partial TC}{\partial Q} = -\frac{1}{Q^2}[s + C(1-t)] + \frac{hE}{2}$$

Equating $\frac{\partial TC}{\partial Q} = 0$, we get,

$$Q = \sqrt{\frac{2[s + C(1-t)]}{hE}}$$

Inventory Model in Fuzzy Sense:

Suppose

$$\breve{s} = (s_1, s_2, s_3)$$

$$\breve{E} = (E_1, E_2, E_3)$$

Are non-negative trapezoidal fuzzy numbers and applying signed distance formula, we get

$$P(\breve{T}C) = \frac{1}{4}\left\{\begin{array}{l} \alpha I + \frac{1}{Q}[s_1 + C(1-t)] + h[P - ER + \frac{E_1Q}{2} + \alpha E(X - W)] + \\ 2\left[\alpha I + \frac{1}{Q}[s_2 + C(1-t)] + h[P - ER + \frac{E_2Q}{2} + \alpha E(X - W)]\right] \\ \alpha I + \frac{1}{Q}[s_3 + C(1-t)] + h[P - ER + \frac{E_3Q}{2} + \alpha E(X - W)] \end{array}\right\}$$

In order to find the minimization of $P(\tilde{T}C)$, the derivative $P(\tilde{T}C)$ of with Q is

$$\frac{\partial P(\tilde{T}C)}{\partial Q} = 0$$

$$\frac{1}{4}\left\{\begin{array}{l} \alpha I + \frac{1}{Q}[s_1 + C(1-t)] + h[P - ER + \frac{E_1Q}{2} + \alpha E(X - W)] + \\ 2\left[\alpha I + \frac{1}{Q}[s_2 + C(1-t)] + h[P - ER + \frac{E_2Q}{2} + \alpha E(X - W)]\right] \\ \alpha I + \frac{1}{Q}[s_3 + C(1-t)] + h[P - ER + \frac{E_3Q}{2} + \alpha E(X - W)] \end{array}\right\} = 0$$

The optimal order quantity

$$Q = \sqrt{\frac{2\{(s_1 + 2s_2 + s_3) + C(1-t)\}}{h(E_1 + 2E_2 + E_3)}}$$

Numerical Analysis:

In a manufacturing factory, the energy consumption and associated costs are carefully monitored to ensure operational efficiency. The factory consumes a total of 1000 units of energy, which helps evaluate its sustainability and energy management practices. The cost of raw materials per batch is 300 currency units, with each batch requiring 220 units of raw material. The production process operates over a short time span of 0.2 hours, and the machines are planned to run over a lifespan of 5 years, considering the absorption coefficient of 0.1. The factory's production rate is 5000 units annually, and the energy rate is set at 20 units per hour, with an energy requirement of 4 units per produced unit. The workload for each process totals 10 hours. The total operational cost, including energy, raw materials, and other expenses, sums up to 7724.76 currency units. The quality control index (Q) is approximately 0.4290, indicating relatively consistent product quality, with standard deviations in production quality ranging between 210, 220, and 230 units, which helps in monitoring variability. Additionally, estimated energy consumption ($\hat{E}$) for future planning falls within the range of 850 to 1150 units, allowing the factory to optimize energy usage and improve overall efficiency. This comprehensive analysis aids in making informed decisions about resource allocation, energy management, and quality assurance to enhance productivity and profitability.

This numerical analysis provides a comprehensive snapshot of the factory's operational performance, encompassing energy consumption, material costs, production efficiency, and quality control. The calculated total cost of 7724.76 currency units, alongside the specific breakdown of input parameters like raw material cost, energy usage, and production rate, offers valuable insights for management. The quality index (Q) of 0.4290, supported by the standard deviation data, indicates a stable production process with predictable output quality. By understanding these metrics, the factory can identify areas for improvement, such as optimizing energy consumption to reduce costs, refining raw material procurement for better efficiency, or implementing targeted quality control measures to further enhance product consistency. Ultimately, this detailed analysis empowers the factory to make data-driven decisions that can lead to increased profitability, sustainable operations, and improved competitive advantage in the manufacturing sector.

Python Code for 3D Surface Plot of Total Cost:

```
importnumpy as np
importmatplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
# Constants (from your data)
C = 300        # Lead time crashing cost
t = 0.2        # Lead time in years
h = 5          # Holding cost
alpha = 0.1    # Fractional cost of capital
i = 5000       # Investment to reduce lost sales
P = 20         # Safety factor
ER = 4         # Lead time per week
X_W = 10       # Expected shortage quantity
# Create grid of shortage cost (s) and demand (E)
s_vals = np.linspace(210, 230, 10)    # Shortage cost from 210 to 230
E_vals = np.linspace(850, 1150, 10)  # Demand from 850 to 1150
S, E = np.meshgrid(s_vals, E_vals)
# Assume order quantity Q = 0.429 (constant as per your model)
Q = 0.429
# Example TC calculation (simplified assumption)
# Let's use a mock total cost function for visualization:
# TC = ordering cost + holding cost + shortage cost + investment cost
TC = (C * E / Q) + (h * Q / 2) + (S * (X_W / E)) + (alpha * i)
# Plotting
fig = plt.figure(figsize=(10, 7))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(S, E, TC, cmap='viridis')
ax.set_title('Total Cost Surface Plot')
ax.set_xlabel('Shortage Cost (s)')
ax.set_ylabel('Demand (E)')
ax.set_zlabel('Total Cost (TC)')
plt.tight_layout()
plt.show()
```

Interpretation of the 3D Surface Plot:

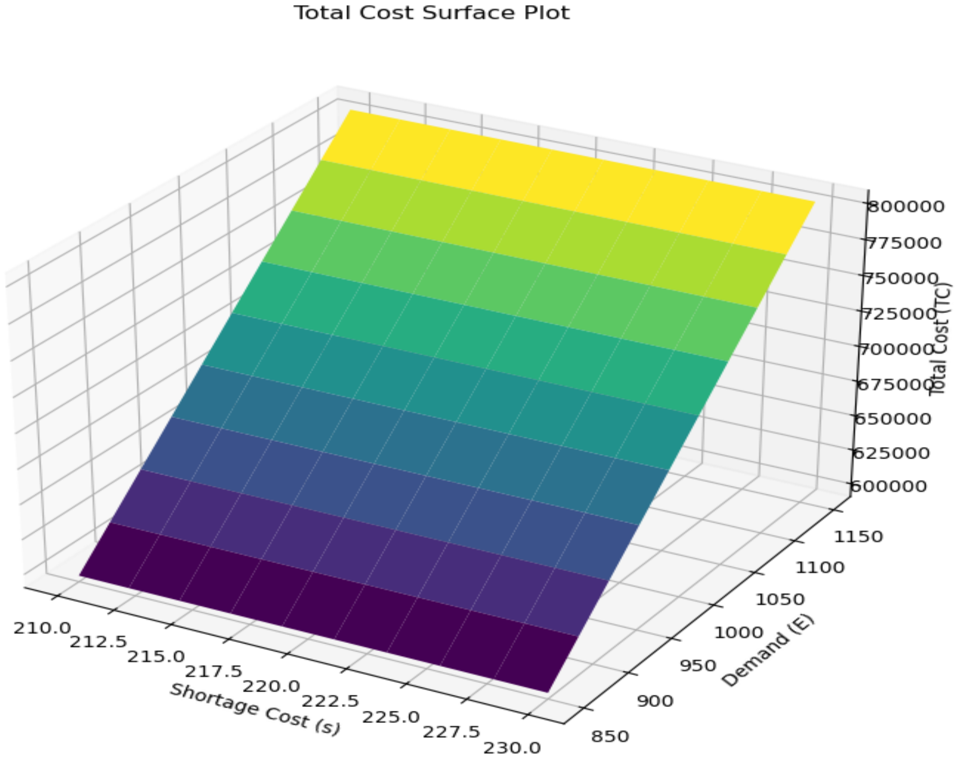

Figure 1

The 3D surface plot illustrates how the total cost (TC) in an inventory system varies with changes in shortage cost (s) and demand (E). The x-axis represents the shortage cost per unit, ranging from 210 to 230. This reflects the penalty incurred for each unit of unmet demand. The y-axis shows the annual demand for the product, varying from 850 to 1150 units. The z-axis represents the total cost, which includes components such as ordering cost, holding cost, shortage penalty, and investment cost to reduce lost sales.

From the plot, it is evident that total cost increases significantly as demand rises. This is intuitive, as higher demand typically requires more frequent ordering, increased inventory storage, and potentially greater investment in reducing stock outs. In contrast, the impact of shortage cost is more gradual. Although the total cost increases with higher shortage penalties, the slope is relatively gentle, suggesting that demand has a stronger influence on total cost than the shortage cost in this model. This is likely because the expected shortage quantity is held constant and small, so even large per-unit penalties do not dramatically affect the overall cost.

The smooth and continuous nature of the surface suggests a stable cost function with no abrupt changes, which is desirable for inventory planning and decision-making. Overall, this visualization helps managers understand how sensitive their total cost is to fluctuations in demand and shortage costs. It supports strategic planning by highlighting the cost implications of rising customer demand and the importance of minimizing stock outs in high-service-level environments.

Conclusion:

This study presented a consistent Economic Order Quantity (EOQ) model under fuzzy conditions, employing the signed distance defuzzification method to effectively bridge the gap between fuzzy and crisp domains. Through the inclusion of fuzzy parameters such as demand, shortage cost, and investment-related variables, the model provides a more flexible and realistic approach to inventory planning in uncertain environments.

The proposed model successfully maintains alignment between fuzzy-based and traditional crisp-based outputs by ensuring that key decision variables particularly order quantity (Q) and total cost (TC) remain consistent across interpretations. The integration of signed distance defuzzification offers mathematical rigor while allowing intuitive interpretations of fuzzy values.

To demonstrate the behaviour of total cost with varying demand and shortage cost parameters, a 3D surface plot was generated using Python. This plot visually confirms that total cost increases significantly with higher demand and more moderately with increased shortage cost. The visualization helps decision-makers understand cost sensitivities in a fuzzy-influenced system and supports more robust inventory policies.

Overall, this work enhances classical EOQ theory by embedding fuzzy logic and defuzzification, offering supply chain managers a practical and reliable tool for decision-making under uncertainty.

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